Consistent supersymmetric Kaluza-Klein truncations with massive modes

JHEP04(2009)102

(http://iopscience.iop.org/1126-6708/2009/04/102)

Published by IOP Publishing for SISSA

Received: *February 2, 2009*
Accepted: *March 10, 2009*
Published: *April 27, 2009*

Consistent supersymmetric Kaluza-Klein truncations with massive modes

Jerome P. Gauntlett, Seok Kim, Oscar Varela and Daniel Waldram

Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.
The Institute for Mathematical Sciences, Imperial College, London SW7 2PE, U.K.
E-mail: j.gauntlett@imperial.ac.uk, s.kim@imperial.ac.uk, o.varela@imperial.ac.uk, d.waldram@imperial.ac.uk

Abstract: We construct consistent Kaluza-Klein reductions of $D = 11$ supergravity to four dimensions using an arbitrary seven-dimensional Sasaki-Einstein manifold. At the level of bosonic fields, we extend the known reduction, which leads to minimal $N = 2$ gauged supergravity, to also include a multiplet of massive fields, containing the breathing mode of the Sasaki-Einstein space, and still consistent with $N = 2$ supersymmetry. In the context of flux compactifications, the Sasaki-Einstein reductions are generalizations of type IIA SU(3)-structure reductions which include both metric and form-field flux and lead to a massive universal tensor multiplet. We carry out a similar analysis for an arbitrary weak G_2 manifold leading to an $N = 1$ supergravity with massive fields. The straightforward extension of our results to the case of the seven-sphere would imply that there is a four-dimensional Lagrangian with $N = 8$ supersymmetry containing both massless and massive spin two fields. We use our results to construct solutions of M-theory with non-relativistic conformal symmetry.

Keywords: AdS-CFT Correspondence, Supergravity Models, Flux compactifications

ArXiv ePrint: 0901.0676

doi:10.1088/1126-6708/2009/04/102

JHEP04(2009)102

Contents

1 Introduction

It is now understood that there are very general situations in which one can perform consistent Kaluza-Klein (KK) reductions of supergravity theories. Starting with any supersymmetric solution of $D = 10$ or $D = 11$ supergravity that is the warped product of an AdS_{d+1} space with an internal space M, it was conjectured [1] (see also [2]) that one can always consistently reduce on the space M to obtain a gauged supergravity theory in $d+1$ dimensions, incorporating only the fields of the supermultiplet containing the metric. In the dual SCFT these fields are dual to the superconformal current multiplet which includes the energy momentum tensor and R symmetry currents. This conjecture has now been proven to be true for a number of general classes of AdS solutions [1, 3–5].

One simple class of examples consists of $AdS_5 \times SE_5$ solutions of type IIB supergravity, dual to $N = 1$ SCFTs in $d = 4$, and $AdS_4 \times SE_7$ solutions of $D = 11$ supergravity, dual to $N = 2$ SCFTs in $d = 3$, where SE_n is an n-dimensional Sasaki-Einstein manifold. In the former case it is known that one can reduce type IIB supergravity on a SE_5 space to get minimal $N = 1$ gauged supergravity in $D = 5$ [3]. Similarly, one can reduce $D = 11$ supergravity on a SE_7 space to get minimal $N = 2$ gauged supergravity in $D = 4$ [1]. In both cases, the bosonic fields in the lower dimensional supergravity theory are massless, consisting of the metric and the gauge field, dual to the energy momentum tensor and the R-symmetry current, respectively. For the special cases when $SE_5 = S^5$ or $SE_7 = S^7$ these truncations were shown to be consistent in [6] and [7], respectively. For these special cases, it is expected or known that there are more general consistent truncations to the maximal gauged supergravities in five dimensions (for various partial results see [8–11]) and four dimensions [12], respectively.

Interestingly, it has recently been shown that the consistent KK reduction of type IIB on a SE_5 space of [3] can be generalised to also include some massive bosonic fields [13]. The bosonic fields included massive gauge fields as well as massive scalars. One of these massive scalars arises from the breathing mode of the SE_5. Viewing the SE_5 space, locally, as a U(1) fibration over a four-dimensional Kähler-Einstein base, the other massive scalar arises from the mode that squashes the size of the fibre with respect to the size of the base. This work thus extends earlier work on including such breathing and squashing modes for the special case of the five-sphere in [14, 15].

In order to understand this in more detail, here we will study similar extensions of the KK reductions of $D = 11$ supergravity on a SE_7 space. For the special case of the seven-sphere some results on KK reductions involving the breathing and squashing modes appear in [14, 15]. In this paper we shall show that one can also generalise the KK reduction of [1] to include massive fields: at the level of the bosonic fields we will show that there is a consistent KK reduction that includes the massless graviton supermultiplet as well as the massive supermultiplet that contains the breathing mode. In the off-shell four-dimensional $N = 2$ theory, in addition to the gravity multiplet, the action contains a tensor multiplet together with a single vector multiplet which acts as a Stückelberg field to give mass to the tensor multiplet. We show that one can also dualize to get an action containing a massive vector multiplet with a gauged hypermultiplet acting as the Stückelberg field. This gives a simple example of the mechanism first observed in [16] and then analyzed in [17–20].

We note that our truncation also has a natural interpretation in terms of flux compactifications. Viewing the SE_7 manifold locally as a U(1) fibration over a Kähler-Einstein manifold, KE_6, one can reduce from M-theory to type IIA. The truncation then has the structure of a IIA reduction on a six-dimensional SU(3) structure manifold [21]. The tensor and vector multiplets in the $N = 2$ action correspond to the universal tensor multiplet which contains the dilaton, the NS two-form B and a complex scalar arising from a RR potential parallel to the $(3, 0)$ form on KE_6, and the universal vector multiplet containing a vector and scalars that arises from scaling the complexified Kähler form. The presence of the background four-form flux, and the "metric fluxes" coming from the twisting of the U(1) fibration and the fact that the $(3, 0)$ form on KE_6 is not closed lead to a gauging of the four-dimensional theory. This is complementary to the model discussed in [22] which had a similar structure but considered different intrinsic torsion in the SU(3) structure. Note that since our truncation is consistent there are no approximations in analysing which KK modes should be kept in the four-dimensional theory.

A simple modification of our ansatz leads to an analogous result for a consistent KK reduction of $D = 11$ supergravity on seven-dimensional manifolds M_7 with weak G_2 holonomy. Recall that such manifolds can be used to construct $AdS_4 \times M_7$ solutions that are dual to $N = 1$ superconformal field theories in $d = 3$. The conjecture of [1] is rather trivial for this case since it just says that there should be a consistent KK reduction to pure $N = 1$ supergravity. Here, however, we will see that this can be extended to include the massive $N = 1$ chiral multiplet that contains the breathing mode of M_7. The consistent KK truncation that we construct is compatible with the general low-energy KK analysis of $D = 11$ supergravity reduced on manifolds with weak G_2 structure that was analysed in [23].

Given these results, it is plausible that for $AdS_4 \times M_7$ solutions with any amount of supersymmetry $1 \leq N \leq 8$ there is a consistent KK truncation that includes both the graviton supermultiplet and the massive breathing mode supermultiplet, preserving all of the supersymmetry. A particularly interesting feature for the case of $N = 8$ supersymmetry, arising from reduction on S^7, is that the supermultiplet containing the breathing mode now contains massive spin-2 fields. Thus if our conjecture is correct the consistent KK reduction would lead to a four-dimensional interacting theory with both massless and massive spin 2 fields, which has been widely thought not to exist. We will return to this point in the discussion section later.

A similar result could also hold for reductions of type IIB on S^5 to maximally supersymmetric theories in five spacetime dimensions containing the massless graviton supermultiplet and the massive breathing mode supermultiplet, which again contains massive spin 2 fields. What is much more certain, however, is that for reductions on SE_5 one can extend the ansatz of [13] to be consistent with $N = 1$ supersymmetry [24].

A principal motivation for constructing consistent KK reductions is that they provide powerful methods to construct explicit solutions. Starting with the work of [25, 26] there has been some recent interest in constructing solutions of string/M-theory that possess a non-relativistic conformal symmetry. In [13] the KK reductions on SE_5 spaces were used to construct such solutions and examples with dynamical exponent $z = 4$ and also $z = 2$, and hence possessing an enlarged Schrödinger symmetry, were found. The solutions with $z = 2$ were independently found in [27, 28]. Here we shall construct similar solutions in $D = 11$ supergravity for arbitrary SE_7 spaces that exhibit a non-relativistic conformal symmetry with dynamical exponent $z = 3$.

Our presentation will focus on supersymmetric $AdS_4 \times SE_7$ solutions. It is well known that for each supersymmetric solution there is a "skew-whiffed" solution obtained by reversing the sign of the four-form flux, or equivalently changing the orientation on the SE_7 [29]. Apart from the special case of the round S^7 the skew-whiffed solution does not preserve any supersymmetry, but is known to be perturbatively stable in supergravity [29]. We will show that for the skew-whiffed solutions there is also a consistent truncation on the SE_7 space to the bosonic fields of a four-dimensional $N = 2$ gauged supergravity theory with an AdS_4 vacuum that uplifts to the skew-whiffed solution. Our action is a non-linear extension of one of those considered recently in [30] in the context of solutions corresponding to holographic superconductivity [31–33] and offers the possibility of finding exact embeddings of such solutions into $D = 11$ supergravity.

JHEP04(2009)102

2 $D = 11$ supergravity reduced on SE$_7$

Our starting point is the class of supersymmetric $AdS_4 \times SE_7$ solutions of $D = 11$ supergravity given by

$$\begin{aligned} ds^2 &= \frac{1}{4} ds^2(AdS_4) + ds^2(SE_7) \\ G_4 &= \frac{3}{8} \operatorname{vol}(AdS_4) \end{aligned} \tag{2.1}$$

where $ds^2(AdS_4)$ is the standard unit-radius metric on AdS_4 and the Sasaki-Einstein metric $ds^2(SE_7)$ is normalised so that the Ricci tensor is six times the metric (as for a unit-radius round seven-sphere). The SE_7 space has a globally defined one-form η that is dual to the Reeb Killing vector, and locally we can write

$$ds^2(SE_7) = ds^2(KE_6) + \eta \otimes \eta \tag{2.2}$$

where $ds^2(KE_6)$ is a local Kähler-Einstein metric with positive curvature, normalised so that the Ricci tensor is eight times the metric. On SE_7 there is also a globally defined two-form J and a $(3, 0)$-form Ω that locally define the Kähler and complex structures on $ds^2(KE_6)$ respectively and satisfy $\Omega \wedge \Omega^* = -8iJ^3/3!$. The Sasaki-Einstein structure implies that

$$\begin{aligned} d\eta &= 2J \,, \\ d\Omega &= 4i\eta \wedge \Omega \,. \end{aligned} \tag{2.3}$$

Our conventions for $D = 11$ supergravity are as in [34]. For completeness, in appendix A we show in detail that given these conventions, together with those for the Sasaki-Einstein structure, the solution (2.1) is indeed supersymmetric.

2.1 The consistent Kaluza-Klein reduction

We now investigate consistent Kaluza-Klein reductions using this class of solutions. Our ansatz for the metric of $D = 11$ supergravity is given by

$$ds^2 = ds_4^2 + e^{2U} ds^2(KE_6) + e^{2V} (\eta + A_1) \otimes (\eta + A_1) \,, \tag{2.4}$$

where ds_4^2 is an arbitrary metric on a four-dimensional spacetime, U and V are scalar fields and A_1 is a one-form defined on the four-dimensional space. For the four-form we take

$$\begin{aligned} G_4 = {} & f \operatorname{vol}_4 + H_3 \wedge (\eta + A_1) + H_2 \wedge J + H_1 \wedge J \wedge (\eta + A_1) \\ & + 2hJ \wedge J + \sqrt{3} \left[\chi_1 \wedge \Omega + \chi (\eta + A_1) \wedge \Omega + \text{c.c.} \right] , \end{aligned} \tag{2.5}$$

where f and h are real scalars, H_p, $p = 1, 2, 3$, are real p-forms, χ_1 is a complex one-form, χ is a complex scalar on the four-dimensional spacetime and "c.c." denotes complex conjugate.

Notice that this ansatz incorporates *all* of the constant bosonic modes that arise from the G-structure tensors (η, J, Ω). It generalises the ansatz considered in [1], as we shall discuss in section 3.1. Together the two scalar fields U and V contain the "breathing mode" of the SE_7 space and the "squashing mode" that scales the fibre direction with respect to the local KE_6 space, as we will discuss more explicitly below. It is also worth observing that if η, J, Ω instead satisfied $d\eta = dJ = d\Omega = 0$ this ansatz would be the same ansatz that one would use to reduce $D = 11$ supergravity on $S^1 \times CY_3$, keeping the universal $N = 2$ vector multiplet, with scalars coming from the volume mode of the Calabi-Yau, and the universal hypermultiplet. In particular, we should expect the same off-shell supermultiplet degrees of freedom to appear in our four-dimensional theory.

We now substitute this ansatz into the equations of motion of $D = 11$ supergravity. We will simply summarise the main results here. More details can be found in appendix B. By analysing the Bianchi identities and the equations of motion for the four-form, we find that the dynamical degrees of freedom turn out to be the four-dimensional fields $g_{\mu\nu}, B_2, B_1, A_1, U, V, h$ and χ with

$$\begin{aligned} H_3 &= dB_2 \\ H_2 &= dB_1 + 2B_2 + hF_2 \\ F_2 &= dA_1 \end{aligned} \tag{2.6}$$

Furthermore we find that $H_1 = dh$ and $\chi_1 = -\frac{i}{4} D\chi$, where

$$D\chi \equiv d\chi - 4iA_1\chi \tag{2.7}$$

and

$$f = 6e^{-6U-V}(1 + h^2 + |\chi|^2) \,. \tag{2.8}$$

Note that the expression for f comes from solving (B.12) and incorporates a convenient integration constant which fixes the radius of the AdS vacuum and also ensures that the reduced $D = 4$ theory includes the supersymmetric $AdS_4 \times SE_7$ solution (2.1). The expression for the four-form can be tidied up a little to read

$$\begin{aligned} G_4 =\ & 6e^{-6U-V}\left(1 + h^2 + |\chi|^2\right) \mathrm{vol}_4 + H_3 \wedge (\eta + A_1) + H_2 \wedge J \\ & + dh \wedge J \wedge (\eta + A_1) + 2hJ \wedge J + \sqrt{3}\left[\chi(\eta + A_1) \wedge \Omega - \frac{i}{4} D\chi \wedge \Omega + \text{c.c.}\right] \,. \end{aligned} \tag{2.9}$$

We find that all dependence on the internal SE_7 space drops out of the $D = 11$ equations of motion and we are left with equations of motion for the four-dimensional fields which are written in appendix B. Thus the ansatz (2.4), (2.9) defines a consistent KK truncation. The equations of motion can be derived from the following four-dimensional action:

$$\begin{aligned} S =& \int d^4x\sqrt{-g}e^{6U+V}\Big[R + 30(\nabla U)^2 + 12\nabla U\cdot\nabla V - \frac{3}{2}e^{-4U-2V}(\nabla h)^2 - \frac{3}{2}e^{-6U}|D\chi|^2 \\ & -\frac{1}{4}e^{2V}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}e^{-2V}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{3}{4}e^{-4U}H_{\mu\nu}H^{\mu\nu} + 48e^{-2U} - 6e^{-4U+2V} \\ & - 24h^2e^{-8U} - 18\left(1 + h^2 + |\chi|^2\right)^2 e^{-12U-2V} - 24e^{-6U-2V}|\chi|^2\Big] \\ & + \int\left[-3hH_2 \wedge H_2 + 3h^2H_2 \wedge F_2 - h^3F_2 \wedge F_2 + 6A_1 \wedge H_3 - \frac{3i}{4}H_3 \wedge (\chi^* D\chi - \chi D\chi^*)\right] \,. \end{aligned} \tag{2.10}$$

JHEP04(2009)102

It is also helpful to write this with respect to the Einstein-frame metric $g_E \equiv e^{6U+V} g$ and we find

$$
\begin{aligned}
S = \int d^4x\sqrt{-g_E}\,\Big[& R_E - 24(\nabla U)^2 - \frac{3}{2}(\nabla V)^2 - 6\nabla U\cdot\nabla V - \frac{3}{2}e^{-4U-2V}(\nabla h)^2 - \frac{3}{2}e^{-6U}|D\chi|^2 \\
& -\frac{1}{4}e^{6U+3V}F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}e^{12U}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{3}{4}e^{2U+V}H_{\mu\nu}H^{\mu\nu} + 48e^{-8U-V} \\
& - 6e^{-10U+V} - 24h^2e^{-14U-V} - 18\left(1+h^2+|\chi|^2\right)^2 e^{-18U-3V} - 24e^{-12U-3V}|\chi|^2\Big] \\
+\int \Big[& -3hH_2\wedge H_2 + 3h^2H_2\wedge F_2 - h^3F_2\wedge F_2 + 6A_1\wedge H_3 - \frac{3i}{4}H_3\wedge(\chi^* D\chi - \chi D\chi^*)\Big] \,.
\end{aligned}
\tag{2.11}
$$

2.2 Masses and dual operators

When we set $H_3 = H_2 = F = U = V = h = \chi = 0$, and thus $f = 6$, the equations of motion are solved by taking the four-dimensional metric to be $\frac{1}{4}ds^2(AdS_4)$. This "vacuum solution" uplifts to give the $AdS_4 \times SE_7$ solution given in (2.1). We can work out the masses of the other fields, considered as perturbations about this vacuum solution, by analysing the quadratic terms in the Lagrangian (2.11). One immediately deduces that the scalar fields h and χ have $m_h^2 = 40$ and $m_\chi^2 = 40$. One can diagonalise the terms involving the scalar fields U and V by writing

$$
\begin{aligned}
U &= -u + \frac{1}{3}v \\
V &= 6u + \frac{1}{3}v
\end{aligned}
\tag{2.12}
$$

and we find that $m_u^2 = 16$ and $m_v^2 = 72$. Note that in terms of u and v our KK ansatz for the metric (2.4) can be written

$$
ds^2 = e^{-7v/3}ds_E^2 + e^{2v/3}\left[e^{-2u}ds^2(KE_6) + e^{12u}(\eta + A_1)\otimes(\eta + A_1)\right] \tag{2.13}
$$

and we can identify the scalar field v as the "breathing mode" and u as the "squashing mode" that squashes the size of the fibre with respect to the size of KE_6, preserving the volume of the SE_7 space.

The quadratic action for the fields A_1, B_1, B_2 (setting $U = V = h = \chi = 0$) is

$$
\int -\frac{1}{2}F_2\wedge *F_2 + \frac{1}{2}H_3\wedge *H_3 - \frac{3}{2}(dB_1 + 2B_2)\wedge *(dB_1 + 2B_2) + 6A_1\wedge H_3 \,. \tag{2.14}
$$

If one ignores the final term, we see that this has the standard form for a massless gauge field A_1 and a massive two-form B_2 with B_1 acting as a Stückelberg field. However the presence of the final term means that the fields are not properly diagonalized. To find the mass eigenstates, it is helpful to regard $H_2' \equiv dB_1$ as a basic field by introducing a Lagrange multiplier one-form $\tilde{B}_1$ and adding a term

$$
\int 3\tilde{B}_1\wedge dH_2' \tag{2.15}
$$

to the action: indeed integrating out $\tilde{B}_1$ brings one back to the original quadratic action. Integrating out H_2' instead, we find $H_2' = - * \tilde{H}_2 - 2B_2$, where $\tilde{H}_2 \equiv d\tilde{B}_1$, and after substitution one obtains the dualised action

$$\int -\frac{1}{2}F_2 \wedge *F_2 + \frac{1}{2}H_3 \wedge *H_3 - \frac{3}{2}\tilde{H}_2 \wedge *\tilde{H}_2 + 6H_3 \wedge (\tilde{B}_1 - A_1) \ . \tag{2.16}$$

Continuing we now introduce

$$\begin{aligned} \mathcal{A}_1 &= \frac{1}{2}\big(A_1 + 3\tilde{B}_1\big) \,, \\ \mathcal{B}_1 &= \frac{\sqrt{3}}{2}\big(A_1 - \tilde{B}_1\big) \,, \end{aligned} \tag{2.17}$$

so that the action can be written

$$\int -\frac{1}{2}d\mathcal{A}_1 \wedge *d\mathcal{A}_1 + \frac{1}{2}H_3 \wedge *H_3 - \frac{1}{2}d\mathcal{B}_1 \wedge *d\mathcal{B}_1 - 4\sqrt{3}H_3 \wedge \mathcal{B}_1 \ . \tag{2.18}$$

Clearly $\mathcal{A}_1$ is a massless vector field. The action for the one-form $\mathcal{B}_1$ and the two-form B_2 appears, for instance, in [35]. It can be viewed as describing either a massive vector or a massive two-form field, which are well-known to be equivalent (see for example [36, 37]), with $m^2 = 48$. For instance, if one further dualises $\mathcal{B}_1$, one obtains the standard Stückelberg form for a massive two-form. Alternatively one can dualise the two-form B_2 to obtain a pseudoscalar a. This is achieved by adding

$$\int a dH_3 \tag{2.19}$$

to the action. Integrating out H_3, we find that $H_3 = - * (da - 4\sqrt{3}\mathcal{B}_1)$ and get the action for a massive vector field $\mathcal{B}_1$

$$\int -\frac{1}{2}d\mathcal{A}_1 \wedge *d\mathcal{A}_1 - \frac{1}{2}d\mathcal{B}_1 \wedge *d\mathcal{B}_1 + \frac{1}{2}\big(da - 4\sqrt{3}\mathcal{B}_1\big) \wedge *\big(da - 4\sqrt{3}\mathcal{B}_1\big) \ . \tag{2.20}$$

In this form, we see that a is a standard Stückelberg scalar field: using the corresponding gauge symmetry to set $a = 0$ reveals that $\mathcal{B}_1$ is indeed massive with $m^2 = 48$.

It is interesting to determine the scaling dimensions of the operators in the dual SCFT that correspond to the modes we are considering. The massless vector field, $\mathcal{A}_1$, has $\Delta = 2$ and the massless graviton has $\Delta = 3$. For the scalar fields, using the formula

$$\Delta = \frac{3}{2} \pm \frac{1}{2}\sqrt{9 + m^2} \tag{2.21}$$

we deduce that the scaling dimensions of u, h, χ and v are given by

$$\Delta_u = 4, \qquad \Delta_h = \Delta_\chi = 5, \qquad \Delta_v = 6 \tag{2.22}$$

Finally, for the massive vector field with $m^2 = 48$, defined by the fields $\mathcal{B}_1$ and B_2, we can use the formula for a massive p-form,

$$\Delta = \frac{3}{2} \pm \frac{1}{2}\sqrt{(3 - 2p)^2 + m^2} \tag{2.23}$$

to deduce that the dual operator has $\Delta = 5$.

We will show in the next section that the fields that we have retained are the bosonic fields of an $N = 2$ supergravity theory. In particular they form the bosonic fields of unitary irreducible representations of $Osp(2|4)$. The KK modes we have kept are present for any Sasaki-Einstein seven-manifold and so, in particular, we can consider the special case of $M(3,2)$ for which the supermultiplet structure was analysed in detail in [38]. The massless graviton and the massless gauge field that we have kept are the bosonic fields of the massless graviton multiplet, whose field content is summarised in table 8 of [38]. By analysing the results of [38] we find[1] that the remaining massive fields are the bosonic fields of a long vector multiplet with field content as in table 3 of [38] with $E_0 = 4$, $y_0 = 0$. Note in particular that with $y_0 = 0$ the only bosonic modes with non-zero R-charge ("hypercharge") are the two scalar fields with $\Delta = 5$. These correspond to the χ fields which indeed have non-zero R-charge since the $(3, 0)$-form Ω in (2.9) carries non-zero R-charge.

2.3 $N = 2$ supersymmetry

We now show that the Lagrangian (2.11) is the bosonic part of an $N = 2$ supersymmetric theory. As formulated it contains, in addition to the $N = 2$ supergravity multiplet, a massive two-form and five scalar fields. The appearance of supersymmetric theories with a massive two-form in dimensional reductions with non-trivial fluxes was first observed in [16]. In terms of supermultiplets the two-form and three scalars should form a tensor multiplet, while the Stückelberg gauge field and the remaining two-scalars form a vector multiplet. The general couplings of such $N = 2$ theories are discussed in [17–19] (see also [20] for the $N = 1$ analogue). For the case in hand, it should be possible to dualize to a massive vector multiplet and a conventional (gauged) hypermultiplet.

As we have noted, our Sasaki-Einstein reduction can also be viewed as a flux compactification of type IIA supergravity. In particular, if instead of a reduction on a Sasaki-Einstein manifold we were considering a reduction on $S^1 \times X$ where X is a Calabi-Yau threefold, the fields U, χ and the scalar dual of B_2 would parameterize a universal hypermultiplet. Similarly, $2U + V$ and h would be the scalars for the universal vector multiplet related to rescaling the metric on the Calabi-Yau space. The kinetic terms of these fields should be unchanged by going to the Sasaki-Einstein reduction, so our expectation is that the action (2.11) can be rewritten as a gauged universal hypermultiplet coupled to a single universal vector multiplet. In the following we will show how this structure arises.

Let us first identify the structure before dualizing. The generic form for the coupling of vector and tensor multiplets has been discussed in some detail for instance in [39]. We note that in identifying with the theory of general $N = 2$ gauged supergravities as summarised in appendix B, we should multiply the overall action in (2.24) by a factor of 1/2, which we will do in this section only. We can write (half) the action (2.11) as

$$S = \int d^4x \sqrt{-g_E} \left(\frac{1}{2} R_E - V \right) + S_V + S_H \tag{2.24}$$

[1] More specifically, our modes are obtained in (3.19) and (3.20) of [38] with $M_1 = M_2 = 0$, and hence $J = 0$, consistent with the fact that the modes are singlets with respect to the SU(3) flavour symmetry of this specific Sasaki-Einstein manifold.

JHEP04(2009)102

where

$$S_V = \frac{1}{2}\int \sqrt{-g_E}\left[-\frac{3}{2}(\nabla(2U+V))^2 - \frac{3}{2}\left(e^{-2U-V}\right)^2(\nabla h)^2 \right.$$
$$\left. -\frac{1}{4}e^{6U+3V}F_{\mu\nu}F^{\mu\nu} - \frac{3}{4}e^{2U+V}H_{\mu\nu}H^{\mu\nu}\right]$$
$$+\frac{1}{2}\int -3hH_2\wedge H_2 + 3h^2 H_2\wedge F_2 - h^3 F_2\wedge F_2\,, \tag{2.25}$$

while

$$S_H = \frac{1}{2}\int d^4x\sqrt{-g_E}\left[-\frac{1}{2}(\nabla(6U))^2 - \frac{1}{12}e^{12U}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{3}{2}e^{-6U}|D\chi|^2\right]$$
$$+\frac{1}{2}\int 6A_1\wedge H_3 - \frac{3i}{4}H_3\wedge(\chi^* D\chi - \chi D\chi^*)\,, \tag{2.26}$$

and

$$V = -24e^{-8U-V} + 3e^{-10U+V} + 12h^2 e^{-14U-V} + 12|\chi|^2 e^{-12U-3V}$$
$$+9\left(1+h^2+|\chi|^2\right)^2 e^{-18U-3V}\ . \tag{2.27}$$

If we ignore the B_2 term in the definition of H_2 in (2.6) we see that S_V can be written in the form of an ungauged vector multiplet action, as summarized in appendix C, as follows. Introducing $\tau = h + ie^{2U+V}$ we define $X^I = (1,\tau)$ and the gauge fields $F^I = \frac{1}{\sqrt{2}}(F_2, -dB_1)$ with $I = 0,1$. One then finds the $\mathcal{N}_{IJ}$ matrix is given by

$$\mathcal{N}_{IJ} = \begin{pmatrix} \tau^2(\tau-3h) & 3h\tau \\ 3h\tau & -3(\tau+h)\end{pmatrix} \tag{2.28}$$

together with the corresponding holomorphic prepotential

$$\mathcal{F}(X) = -\frac{(X^1)^3}{X^0}, \tag{2.29}$$

giving the Kähler potential

$$K_V = -\log\left(i\bar{X}^I\mathcal{F}_I - iX^I\bar{\mathcal{F}}\right) = -\log i(\tau-\bar{\tau})^3\ . \tag{2.30}$$

This is the standard form that arises from flux compactification on a SU(3) structure manifold [16], with a single Kähler modulus. We also note that, if we ignore the coupling to the vector multiplets, one can dualize the two-form B_2 to get a pseudo-scalar a by adding the term (2.19) to S_H giving $e^{12U}H_3 = -*\left[da - \frac{3i}{4}(\chi^* d\chi - \chi d\chi^*)\right]$. Then identifying $\rho = 4e^{6U}$, $\sigma = 4a$ and $\xi = \sqrt{3}\bar{\chi}$, we get the standard metric (C.9) on the universal hypermultiplet space.

To make the full dualization from the massive tensor multiplet $(e^{6U}, B_2, \chi, \bar{\chi})$ to a massive vector multiplet, one must first dualise the field B_1 by adding the term (2.15) to the action and then integrating out H_2'. We now find that

$$H_2' + 2B_2 + hF_2 = \frac{1}{4h^2 + e^{4U+2V}}\left[2h(\tilde{H}_2 + h^2F_2) - e^{2U+V} * (\tilde{H}_2 + h^2 F_2)\right] \tag{2.31}$$

where $\tilde{H}_2 \equiv d\tilde{B}_1$ as before, and after substitution one finds a dual action containing gauge fields $A_1, \tilde{B}_1, B_2$. One can then dualise the two-form B_2 to obtain a pseudo-scalar by adding the term (2.19). After integrating out H_3 we now find

$$e^{12U} H_3 = - * \left[da - 6(A_1 - \tilde{B}_1) - \frac{3i}{4}(\chi^* D\chi - \chi D\chi^*) \right] \,. \tag{2.32}$$

After these dualisations the new expressions for S_V and S_H are

$$\begin{aligned} S_V = & \frac{1}{2}\int d^4x\sqrt{-g_E}\left[-\frac{3}{2}(\nabla(2U+V))^2 - \frac{3}{2}\left(e^{-2U-V}\right)^2(\nabla h)^2\right] \\ & +\frac{1}{2}\int\left[\frac{3}{2}\,\mathrm{Im}(\tau+h)^{-1}\big(\tilde{H}_2+h^2F_2\big)\wedge *\big(\tilde{H}_2+h^2F_2\big)\right. \\ & \qquad +\frac{3}{2}\,\mathrm{Re}(\tau+h)^{-1}\big(\tilde{H}_2+h^2F_2\big)\wedge\big(\tilde{H}_2+h^2F_2\big) \\ & \qquad \left. -\frac{1}{2}\big(e^{2U+V}\big)^3F_2\wedge *F_2 - 3h\tilde{H}_2\wedge F_2 - h^3F_2\wedge F_2\right] \end{aligned} \tag{2.33}$$

and

$$\begin{aligned} S_H = & -\frac{1}{4}\int d^4x\sqrt{-g_E}\Big[\big(\nabla(6U)\big)^2 + 3(e^{-6U})\big|d\chi - 4iA_1\chi\big|^2 \\ & \qquad + \big(e^{-6U}\big)^2\left(\nabla a - 6(A_1-\tilde{B}_1) - \frac{3i}{4}(\chi^*D\chi - \chi D\chi^*)\right)^2\Big] \,. \end{aligned} \tag{2.34}$$

We now compare S_V with the general gauged $N = 2$ action (C.1) given in appendix C. If we identify the gauge fields $\tilde{F}^I = (F_2, -\tilde{H}_2)$ and introduce new homogeneous coordinates $\tilde{X}^I = (1, \tau^2)$, we find the gauge kinetic matrix in (C.1) is given by

$$\tilde{\mathcal{N}}_{IJ} = \frac{1}{2(\tau+h)}\begin{pmatrix} -\tau^3\bar{\tau} & 3h\tau \\ 3h\tau & 3 \end{pmatrix} \,, \tag{2.35}$$

and, since we have dualized the gauge fields, there is a new holomorphic prepotential

$$\tilde{\mathcal{F}} = \sqrt{\tilde{X}^0(\tilde{X}^1)^3} \,. \tag{2.36}$$

This indeed correctly reproduces (2.35) and leads to the Kähler potential

$$\tilde{K}_V = -\log i(\tau-\bar{\tau})^3 + \log 2 \,, \tag{2.37}$$

which agrees with (2.30) up to a (constant) Kähler transformation. Notice that $(\tilde{X}^I, \tilde{\mathcal{F}}_I)$ and $\tilde{\mathcal{N}}_{IJ}$ can be obtained from $(X^I, \mathcal{F}_I)$ and $\mathcal{N}_{IJ}$ by a symplectic transformation (C.5) with

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \sqrt{2}\left(\begin{array}{cc|cc} 1 & & & \\ & 0 & & -\frac{1}{3} \\ \hline & & \frac{1}{2} & \\ & \frac{3}{2} & & 0 \end{array}\right) \,, \tag{2.38}$$

together with a rescaling of the $\tilde{X}^I$ homogeneous coordinates by $1/\sqrt{2}$. Note that, up to a normalization and as expected given we are dualizing B_1, the matrix (2.38) simply exchanges the electric and magnetic gauge fields for F^1.

JHEP04(2009)102

Now we consider S_H. Identifying $\rho = 4e^{6U}$, $\sigma = 4a$, $\xi = \sqrt{3}\bar{\chi}$ we see that it indeed matches the universal hypermultiplet form given in (C.9). In appendix C we have labelled these coordinates q^u, $u = 1, \ldots, 4$. From the terms $Dq^u = dq^u - k^u_I A^I$ in (C.1) we see that gauging is along Killing vectors

$$\begin{aligned} k_0 &= 6\partial_a + 4i(\chi\partial_\chi - \bar{\chi}\partial_{\bar{\chi}}) = 24\partial_\sigma - 4i(\xi\partial_\xi - \bar{\xi}\partial_{\bar{\xi}})\,, \\ k_1 &= 6\partial_a = 24\partial_\sigma\ . \end{aligned} \tag{2.39}$$

Given the formulae in appendix C for the quaternionic geometry it is straightforward to calculate that for the Killing vector $k = \partial_\sigma$ we have the Killing prepotential

$$P_\sigma = \begin{pmatrix} i/4\rho & 0 \\ 0 & -i/4\rho \end{pmatrix} \tag{2.40}$$

and for $k = i\xi\partial_\xi - i\bar{\xi}\partial_{\bar{\xi}}$ we have

$$P_\xi = \begin{pmatrix} \frac{i}{2}(1-\rho^{-1}\xi\bar{\xi}) & -i\xi\rho^{-1/2} \\ -i\bar{\xi}\rho^{-1/2} & -\frac{i}{2}(1-\rho^{-1}\xi\bar{\xi}) \end{pmatrix}\ . \tag{2.41}$$

The Killing prepotentials P_I, corresponding to (2.39), are therefore

$$P_0 = 24P_\sigma - 4P_\xi\,, \qquad P_1 = 24P_\sigma\ . \tag{2.42}$$

Finally substituting these expressions into the general form (C.8) for the potential V we reproduce (2.27). This completes our demonstration that our action is the bosonic action of an $N = 2$ supergravity theory

3 Some further truncations

In this section we observe that there are some additional consistent truncations incorporated in our KK ansatz (2.4), (2.9), compatible with the general equations of motion contained in appendix B. We begin by observing that it is consistent to set the complex scalar field $\chi = 0$. This is not surprising as this is the only field in the ansatz that carries non-zero R-charge. The resulting equations of motion can be obtained from an action obtained by setting $\chi = 0$ in (2.11).

3.1 Minimal gauged supergravity

It is also consistent to set $U = V = h = \chi = H_3 = 0$, $f = 6$ and $H_2 = -*F_2$. This sets all of the massive fields to zero and we then find that the equations of motion come from a Lagrangian given by

$$S = \int d^4x\sqrt{-g}\,[R - F_{\mu\nu}F^{\mu\nu} + 24] \tag{3.1}$$

This is the consistent KK reduction on a SE_7 to the massless fields of $N = 2$ $D = 4$ gauged supergravity that was discussed in [1].

It is interesting to ask whether this truncation to minimal gauged supergravity can be extended to just include the breathing mode scalar v. However, if we take $h = \chi = H_3 = 0$, $H_2 = -*F_2$ with $U = V = v/3$ and $f = 6e^{-7v/3}$ we find that consistency requires $v = 0$ in addition.

JHEP04(2009)102

3.2 Scalars

We next observe that it is possible to consistently truncate to just scalar fields plus the metric by setting $H_3 = H_2 = F_2 = 0$ and $\mathrm{Im}\,\chi = 0$. The resulting equations of motion follow from an action which can be obtained by substituting this truncation directly into the general action (2.11):

$$S = \int d^4x\sqrt{-g_E}\Big[R_E - 42(\nabla u)^2 - \frac{7}{2}(\nabla v)^2 - \frac{3}{2}e^{-8u-2v}(\nabla h)^2 - \frac{3}{2}e^{6u-2v}(\nabla\chi_{\mathrm{R}})^2 + 48e^{2u-3v} \\ -6e^{16u-3v} - 24h^2e^{8u-5v} - 18(1+h^2+\chi_{\mathrm{R}}^2)^2e^{-7v} - 24e^{-6u-5v}\chi_{\mathrm{R}}^2\Big] \quad (3.2)$$

and we have switched from U and V to u and v via (2.12) and $\chi_{\mathrm{R}} = \mathrm{Re}\,\chi$.

In fact, it is also consistent to further set $\chi_{\mathrm{R}} = 0$ or $h = 0$, or both, and the equations of motion are those that are obtained by substituting into the action (3.2). Note that for the case of the seven-sphere setting $\chi_{\mathrm{R}} = h = 0$, which just maintains the breathing and squashing mode scalars, was also considered[2] in [14]. Indeed if we substitute $\chi_{\mathrm{R}} = h = 0$, $v = -\tilde{\varphi}/\sqrt{7}$ and $u = \tilde{\phi}/2\sqrt{21}$ into (3.2) we obtain results equivalent to (2.20) and (2.21) of [14].

A different, further consistent truncation is achieved by setting $u = 0$ and $\chi_{\mathrm{R}} = \frac{2}{\sqrt{3}}h$ in (3.2). This truncation generically breaks supersymmetry down to $N = 1$, as we will see in the next subsection.

3.3 The weak G_2 case

As we have just noted, it is consistent to set $H_3 = H_2 = F_2 = 0$, $u = 0$ (or, equivalently, $U = V$) and $\chi = \frac{2}{\sqrt{3}}h$. The resulting equations of motion can be obtained from an action which can be obtained from (3.2) and reads:

$$S = \int d^4x\sqrt{-g_E}\Big[R_E - \frac{7}{2}(\nabla v)^2 - \frac{7}{2}e^{-2v}(\nabla h)^2 + 42e^{-3v} - 56e^{-5v}h^2 - 2\left(3+7h^2\right)^2 e^{-7v}\Big]. \quad (3.3)$$

Note that expanding about the AdS_4 vacuum we find $m_v^2 = 72$, $m_h^2 = 40$ and hence $\Delta_v = 6$, $\Delta_h = 5$.

It is interesting to observe that, for this truncation, the KK ansatz (2.4), (2.9) for the $D = 11$ fields can be written

$$\begin{aligned} ds^2 &= ds_4^2 + e^{2v/3}ds^2(SE_7) \\ G_4 &= f\,\mathrm{vol}_4 + dh\wedge\varphi + 4h *_7 \varphi \end{aligned} \quad (3.4)$$

where $f = 2e^{-7v/3}(3+7h^2)$ and we have introduced the quantities

$$\begin{aligned} \varphi &= J\wedge\eta + \mathrm{Im}\,\Omega \\ *_7\varphi &= \frac{1}{2}J\wedge J + \eta\wedge\mathrm{Re}\,\Omega \end{aligned} \quad (3.5)$$

that satisfy

$$d\varphi = 4 *_7 \varphi \quad (3.6)$$

[2] Note that in section 2.2 of [14] they also consider the truncation with, in the language of this paper, $h = \chi = H_2 = 0$. However, this is not a consistent truncation: equation (B.4) implies that $H_3 = 0$ and then (B.9) implies that $F_2 = 0$.

Interpreting φ as a G_2 structure on the seven-dimensional space SE_7, the condition (3.6) is equivalent to weak G_2 holonomy (i.e. that the cone over the space has Spin(7) holonomy). One can then generalize by replacing SE_7 with an arbitrary space M with weak G_2 holonomy and the ansatz (3.4) still gives a consistent truncation. One would expect such a truncation to have $N=1$ supersymmetry, with the metric lying in an $N=1$ supermultiplet and the breathing mode in a massive $N=1$ chiral multiplet. In fact, introducing a complex scalar $\phi = e^v + ih$, the $N=1$ supersymmetry of the action (3.3) can be explicitly exhibited by rewriting it in terms of a Kähler metric $g_{\phi\bar\phi} = \partial_\phi \partial_{\bar\phi} K$ with Kähler potential $K = -7\log(\phi+\bar\phi)$, and a superpotential $W = 4\sqrt{2}(7\phi^2 - 3)$, as

$$S = \int d^4x\sqrt{-g_E}\left[R_E - 2g_{\phi\bar\phi}\partial_\mu\phi\partial^\mu\bar\phi - 2e^K\left(g^{\phi\bar\phi}|D_\phi W|^2 - 3|W|^2\right)\right] \tag{3.7}$$

where $D_\phi W = \partial_\phi W + (\partial_\phi K)W$. It is worthwhile noting that starting with the KK ansatz (3.4) this superpotential can be derived from the general expression for the form of the superpotential in KK reductions on manifolds with G_2 structure that was obtained in [23]. In contrast to [23], here we have also shown that this particular KK reduction is a consistent KK truncation.

We could go one step further and also set $h=0$. We then get the consistent KK truncation that is valid for any Einstein seven-manifold, where one keeps only the metric and the breathing mode scalar. The action is given by

$$S = \int d^4x\sqrt{-g_E}\left[R_E - \frac{7}{2}(\nabla v)^2 + 42e^{-3v} - 18e^{-7v}\right] \tag{3.8}$$

and we see that $m_v^2 = 72$ and hence $\Delta_v = 6$. The action (3.8) was first obtained in [14] in the context of the seven-sphere. Specifically (3.8) can be obtained from (2.6), (2.7) of [14] by setting $\phi = -\sqrt{7}v$, $c=6$, $R_7 = 42$.

3.4 Massive vector

We can also consider a truncation to a metric and a massive vector field. We now set $U = V = h = \chi = 0$, $f=6$,$H_2 = \frac{1}{3} * F_2$ and $H_3 = 8 * A_1$. We now find that *provided we restrict to configurations that satisfy*

$$F_2 \wedge F_2 = F_2 \wedge *F_2 = A_1 \wedge *A_1 = 0\,, \tag{3.9}$$

the equations of motion can be written

$$\begin{aligned} d*F_2 &= 48 * A_1\,, \\ R_{\mu\nu} &= -12g_{\mu\nu} + \frac{2}{3}F_{\mu\rho}F_\nu{}^\rho + 32A_\mu A_\nu \\ &= -12g_{\mu\nu} + \frac{2}{3}\left(F_{\mu\rho}F_\nu{}^\rho - \frac{1}{8}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right) + 32\left(A_\mu A_\nu - g_{\mu\nu}A_\rho A^\rho\right) \end{aligned} \tag{3.10}$$

where we have used (3.9) to get the last line. These equations of motion come from the Lagrangian

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{3}F_{\mu\nu}F^{\mu\nu} - 32A_\mu A^\mu + 24\right] \tag{3.11}$$

JHEP04(2009)102

which describes a metric coupled to a massive vector field with $m^2 = 48$, provided that we impose the conditions (3.9) by hand. We will return to this truncation to construct solutions of $D = 11$ supergravity in the next section.

4 Solutions

As an application we construct solutions of $D = 11$ supergravity by constructing solutions to the four-dimensional equations given in (3.9) and (3.10). We consider the ansatz given by

$$\begin{aligned} ds^2 &= -\alpha^2\rho^{2k}(dx^+)^2 + \frac{d\rho^2}{4\rho^2} + \frac{\rho^2}{4}\left(-dx^+dx^- + dx^2\right) \\ A_1 &= c\rho^k dx^+ \end{aligned} \tag{4.1}$$

We find that $k = 3$ with $c^2 = \alpha^2$ solves all the equations as does $k = -4$ with $c^2 = 15\alpha^2/8$. We can now uplift these solutions to $D = 11$ by setting $U = V = h = \chi = 0$, $f = 6$, $H_2 = \frac{1}{3} * F$ and $H_3 = 8 * A_1$ and substituting into (2.4) and (2.9). Writing this out explicitly for the $k = 3$ case case we obtain

$$\begin{aligned} ds^2 &= -\alpha^2\rho^6(dx^+)^2 + \frac{d\rho^2}{4\rho^2} + \frac{\rho^2}{4}\left(-dx^+dx^- + dx^2\right) + ds^2(KE_6) + (\eta + \alpha\rho^3 dx^+)^2 \\ G &= \frac{3}{16}\rho^2 dx^+ \wedge dx^- \wedge d\rho \wedge dx + \frac{1}{2}\alpha\, dx^+ \wedge dx \wedge d(\rho^4\eta) \end{aligned} \tag{4.2}$$

This solution is in close analogy to the solutions considered in [13] and has a non-relativistic conformal symmetry with dynamical exponent $z = 3$ i.e. is invariant under Galilean transformations generated by time and spatial translations, Galilean boosts, a central mass operator, and scale transformations.[3] This solution is supersymmetric, generically preserving two supersymmetries, as explained in [40].

5 Skew-whiffing

Recall that for each $AdS_4 \times M_7$ Freund-Rubin solution there is another "skew-whiffed" solution [29] which can be obtained by reversing the sign of the flux (or equivalently changing the orientation of M_7). With the exception of the special case where M_7 is the round S^7, at most only one of the two solutions is supersymmetric. For example, if we reverse the sign of the flux in the supersymmetric $AdS_4 \times SE_7$ solution (2.1) we obtain another $AdS_4 \times SE_7$ solution of $D = 11$ supergravity given by

$$\begin{aligned} ds^2 &= \frac{1}{4}ds^2(AdS_4) + ds^2(SE_7) \\ G_4 &= -\frac{3}{8}\,\mathrm{vol}(AdS_4) \end{aligned} \tag{5.1}$$

which does not preserve any supersymmetry (provided SE_7 is not S^7).

[3]Recall that only for dynamical exponent $z = 2$ can the algebra be enlarged to include an additional special conformal generator. Also note that the $k = -4$ solution has a non-relativistic conformal symmetry with dynamical exponent $z = -4$ and is singular. We note analogous singular solutions also exist for the theory considered in [13].

By a very small modification of the truncation discussed in section 2.1 above, we can obtain a second consistent truncation on SE_7 to a $D = 4$ theory that contains the skew-whiffed solution. In particular, we solve (B.12) by now setting

$$f = 6e^{-6U-V}(-1 + h^2 + |\chi|^2) \tag{5.2}$$

where we have changed the sign of the constant factor (when $U = V = h = \chi = 0$). The rest of the analysis essentially goes through unchanged but the sign propagates into the $D = 4$ action in two places. In (2.11) $\left(1 + h^2 + |\chi|^2\right)^2 \to \left(-1 + h^2 + |\chi|^2\right)^2$ and $6A_1 \wedge H_3 \to -6A_1 \wedge H_3$. The AdS_4 vacuum solution of this theory now uplifts to the non-supersymmetric skew-whiffed solution. The mass spectrum for this vacuum can be easily calculated and the only difference from section 2.2 is that now $m_\chi^2 = -8$ and $m_h^2 = -8$ corresponding to operators with $\Delta_\pm = 1, 2$. As expected this bosonic mass spectrum is inconsistent with a vacuum preserving $N = 2$ $D = 4$ supersymmetry since it does not match the bosonic $Osp(2|4)$ multiplet structure.

Despite the fact that the skew-whiffed vacuum is not supersymmetric the $D = 4$ action has the bosonic content consistent with $N = 2$ supersymmetry. The analysis of section 2.3 goes through essentially unchanged, but the sign change in the $D = 4$ action, $6A_1 \wedge H_3 \to -6A_1 \wedge H_3$, means that the gauging is now along Killing vectors given by

$$\begin{aligned} k_0 &= -6\partial_a + 4i(\chi\partial_\chi - \bar{\chi}\partial_{\bar{\chi}}) = -24\partial_\sigma - 4i(\xi\partial_\xi - \bar{\xi}\partial_{\bar{\xi}}) \,, \\ k_1 &= 6\partial_a = 24\partial_\sigma \ . \end{aligned} \tag{5.3}$$

The corresponding Killing prepotentials P_I are then

$$P_0 = -24P_\sigma - 4P_\xi \,, \qquad P_1 = 24P_\sigma \ . \tag{5.4}$$

Substituting these expressions into the general form (C.8) for the potential V we reproduce (2.27) after the change $\left(1 + h^2 + |\chi|^2\right)^2 \to \left(-1 + h^2 + |\chi|^2\right)^2$. For a general SE_7 the AdS_4 vacuum spontaneously breaks the $N = 2$ supersymmetry[4] of the action with $f = -6$.

As we have already noted, for the special case that SE_7 is the round S^7, the corresponding $AdS_4 \times S^7$ solutions are supersymmetric for either sign of the flux. It is interesting to observe that while the AdS_4 vacuum of the truncated theory with $f = 6$ contains modes that fall into $OSp(2|4)$ multiplets, this is not the case for the AdS_4 vacuum of the theory with $f = -6$, despite the fact that the uplifted solution is (maximally) supersymmetric. In particular, while the $f = 6$ theory retains an $N = 2$ breathing mode multiplet together with the supergravity multiplet, in the $f = -6$ theory the modes corresponding to the h and χ fields are no longer part of the breathing multiplet but instead are part of the $N = 8$ graviton supermultiplet. Nonetheless this leads to a consistent truncation. This is a novel and interesting phenomenon that would be worth investigating further, including from the dual SCFT point of view.

Many of the additional truncations of the $N = 2$ theory that we considered in section 3 have similar analogues in the skew-whiffed theory with only some minor obvious sign

[4]Here we are assuming that the truncation at the level of the bosonic fields can be extended to include the fermions.

changes required. For example, the $D = 4$ action that contains the non-supersymmetric skew-whiffed weak G_2 case can be written in a manifestly $N = 1$ language and we find that the only difference is that the superpotential $W = 4\sqrt{2}(7\phi^2 - 3) \to 4\sqrt{2}(7\phi^2 + 3)$. For the reduction to the massive vector field, we should now set $U = V = h = \chi = 0$, $f = -6$, $H_2 = -\frac{1}{3} * F_2$ and $H_3 = -8 * A_1$. These sign changes mean that when we uplift the solution (4.1) to $D = 11$ we obtain the solution (4.2) but with the sign of the four-form flux reversed. Note however, as noticed in [30], it is no longer possible to truncate to the field content of minimal gauged supergravity as in section 3.1.

Recently KK reductions of $AdS_4 \times SE_7$ solutions were considered at the linearised level [30] and it was shown that, for the skew-whiffed solution, the modes corresponding to the massless gauge-field $\mathcal{A}_1$ and the complex scalar χ lead to a $D = 4$ theory that exhibits holographic superconductivity. Indeed, at the linearised level, in our analysis we can set $U = V = h = H_3 = 0$ with $F_2 = \pm * H_2$ where the upper (lower) sign corresponds to the supersymmetric (skew-whiffed) truncation. Writing $A_1 = \mathcal{A}/2$, $\chi = \sqrt{2/3}\phi$, the linearised action is given by

$$S = \int d^4x\sqrt{-g}\left[R + 24 - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - |D\phi|^2 - m^2|\phi|^2\right] \tag{5.5}$$

with $\mathcal{F} = d\mathcal{A}$, $D\phi = d\phi - 2i\mathcal{A}\phi$ and $m^2 = 40, -8$ for the supersymmetric and skew-whiffed case, respectively. This is in agreement with [30], upon setting $M^2 = 2$, $L^2 = 1/4$, $q = 2$ and $g = 1$ in their equation (1). In particular, for the skew-whiffed solution, there are solutions of this linearised theory corresponding to holographic superconductors. Our generalised, non-linear and consistently truncated action for the skew-whiffed solutions thus provides an ideal set up to extend the work of [30] to obtain analogous exact solutions of $D = 11$ supergravity.

6 Discussion

In this paper we have considered consistent truncations on Freund-Rubin backgrounds, keeping the breathing mode and with varying degrees of supersymmetry. We have shown that for $AdS_4 \times M_7$ solutions of $D = 11$ supergravity where M_7 is an Einstein space, it is always consistent to truncate the KK spectrum to the graviton plus the breathing mode, which is dual to an operator in the dual CFT with $\Delta = 6$. For $AdS_4 \times M_7$ solutions with $N = 1$ and $N = 2$ supersymmetry, where M_7 has weak G_2 holonomy or is a Sasaki-Einstein seven manifold, respectively, we have also shown that it is consistent to truncate to the massless graviton supermultiplet combined with the supermultiplet containing the massive breathing mode. In both cases, the KK ansatz contains the constant KK modes associated with the weak G_2 or the Sasaki-Einstein structure.

Moving to $AdS_4 \times M_7$ solutions with $N = 3$ supersymmetry, where M_7 is tri-Sasakian, it is natural to expect that a similar story unfolds. Recall that a tri-Sasakian manifold has an SO(3) group of isometries corresponding to SO(3) R-symmetry. By writing down a KK ansatz that incorporates the constant modes associated with the tri-Sasaki structure we strongly suspect that it will be possible to obtain a consistent KK truncation with

$N = 3$ supersymmetry. Such a truncation would retain the fields of the massless graviton supermultiplet (table 3 of [41]) which consist of the graviton and the SO(3) vector fields, and the breathing mode supermultiplet, which now sits in a long gravitino multiplet (table 2 of [41] with $J_0 = 0$) consisting of six massive vectors, transforming in two spin-one representations of SO(3), four scalars in the spin-zero representation, and ten scalars transforming in two spin-two representations.

Following this pattern one is led to consider the maximally supersymmetric $AdS_4 \times S^7$ solution with $N = 8$ supersymmetry. It is again natural to conjecture that there is an analogous consistent KK truncation that extends the one containing just the $N = 8$ graviton supermultiplet [12], i.e. $N = 8$ SO(8) gauged supergravity, to also include the $N = 8$ supermultiplet containing the breathing mode. Using the results of [42] or [43] we conclude that the bosonic fields of this supermultiplet consist of scalars in the $\mathbf{294_v}$, $\mathbf{840'_s}$, $\mathbf{300}$, $\mathbf{35_s}$ and $\mathbf{1}$ irreps of SO(8), where the singlet is the breathing mode, vectors in the $\mathbf{567_v}$, $\mathbf{350}$ and $\mathbf{28}$ irreps and massive spin-two fields in the $\mathbf{35_v}$ irrep. A particularly interesting feature is the appearance of massive spin-two fields in addition to the graviton. This is remarkable since some general arguments have been put forward, for instance in [44], that it is not possible to have consistent theories of a finite number of massive and massless spin-two fields. However, for instance, the group theory arguments in [44], as for conventional $N = 8$ SO(8) supergravity, are not directly applicable here, and furthermore we are led to a theory with a very particular matter content, which suggests a picture where consistency arises from particular conspiracies among the fields, and perhaps depending crucially on the existence of an AdS vacuum. If this putative theory exists, it may also not be possible to further truncate the theory while keeping massive spin-two fields. It is worth pointing out that unlike the cases we have studied in this paper, and the tri-Sasakian case mentioned above, it is much less clear how to directly construct the KK truncation ansatz for this case.

Let us now return to the $AdS_5 \times M_5$ solutions of type IIB supergravity where M_5 is Einstein. Once again there is a consistent KK truncation that keeps the graviton and the breathing mode which is now dual to an operator with $\Delta = 8$. If M_5 is Sasaki-Einstein then it is possible to generalise the ansätze of [3] and of [13] to obtain a consistent KK truncation that includes the bosonic fields of the $N = 1$ graviton multiplet plus the breathing mode multiplet. We will report on the details of this in [24].

For the special case when $M_5 = S^5$ we are led to conjecture that there is a consistent truncation to the massless graviton supermultiplet, i.e. the fields of maximal SO(6) gauged supergravity, combined with the massive breathing mode multiplet whose field content can be obtained from [45]: the bosonic fields consist of scalars in the $\mathbf{105}$, $\mathbf{126_C}$, $\mathbf{20_C}$, $\mathbf{84}$, $\mathbf{10_C}$, $\mathbf{1}$ irreps of SU(4), where the breathing mode is again the singlet, vectors in the $\mathbf{175}$, $\mathbf{64_C}$, $\mathbf{15}$ irreps, two-forms in the $\mathbf{6_C}$, $\mathbf{45_C}$, $\mathbf{50_C}$ irreps and massive spin-two fields in the $\mathbf{20}$ irrep. Note that for this case the operator dual to the breathing mode has been argued to be dual to an operator in $N = 4$ super Yang-Mills theory of the form $TrF^4 + \ldots$, where here F is the $N = 4$ Yang-Mills field strength, and it has been argued that its detailed form can be obtained from expanding the Dirac-Born-Infeld action for the D3-brane [46–48].

In a similar spirit we can consider $AdS_7 \times S^4$ solutions of $D = 11$ supergravity. There is a known consistent truncation [14] that keeps the graviton and the breathing mode which is now dual to an operator with scaling dimension $\Delta = 12$. If this can be extended to include the full $N = 8$ supermultiplets then there would be a consistent KK truncation extending the known one to maximal SO(5) gauged supergravity [49, 50] to also include the breathing mode supermultiplet. The field content of this latter multiplet can be found in [51] (based on the results of [52, 53]): we find scalars in the **55**, **35** and **1** irreps of SO(5), where the singlet is the breathing mode, vectors in the **81** and **10** irreps, three-forms satisfying self-dual equations in the **30** and **5** irreps, two-forms in the **35** irrep and massive spin-two fields in the **14** irrep.

It would also be interesting to see if similar results can be obtained for classes of supersymmetric AdS solutions outside of the Freund-Rubin class that we have been considering so far. The KK truncations to the massless graviton supermultiplets for the class of $N = 2$ and $N = 1$ AdS_5 solutions of $D = 11$ supergravity classified in [54] and [55] were presented in [5] and [4], respectively. Similarly, the KK truncations for the class of $N = 2$ AdS_4 solutions of $D = 11$ supergravity and the class of $N = 1$ AdS_5 solutions of type IIB which were classified in [56] and [57], respectively, were presented in [1]. It would be interesting to extend these KK truncations to also include breathing mode multiplets.

Finally, it would be desirable to have an argument from the SCFT side of the correspondence as to why the KK truncations containing both the graviton multiplets and the massive breathing mode multiplets are consistent.

Acknowledgments

We would like to thank Frederik Denef, Mike Duff, Ami Hanany, Sean Hartnoll, Chris Hull, Mukund Rangamani, James Sparks, Kelly Stelle, Arkady Tseytlin and Toby Wiseman for helpful discussions. JPG is supported by an EPSRC Senior Fellowship and a Royal Society Wolfson Award. OV is supported by a Spanish Government's MEC-FECYT postdoctoral fellowship, and partially through MEC grant FIS2008-1980.

A Supersymmetry of the $AdS_4 \times SE_7$ solution

In this appendix we show that the solution given by

$$\begin{aligned} ds^2 &= \frac{1}{4} ds^2(AdS_4) + ds^2(SE_7)\,, \\ G_4 &= 6\,\mathrm{vol}_4 = \frac{3}{8}\,\mathrm{vol}(AdS_4)\,, \end{aligned} \tag{A.1}$$

is supersymmetric given our set of conventions. These are that of [34] for $D = 11$ supergravity, that the structure on SE_7 is defined by the forms η, J and Ω satisfying

$$\begin{aligned} d\eta &= 2J\,, \\ d\Omega &= 4i\eta\wedge\Omega\,, \\ \mathrm{vol}(SE_7) &= \eta\wedge\frac{1}{3!}J^3 = \eta\wedge\frac{i}{8}\Omega\wedge\Omega^*\,, \end{aligned} \tag{A.2}$$

and that the $D = 11$ volume form is $\epsilon = \mathrm{vol}_4 \wedge \mathrm{vol}(SE_7)$.

It will be sufficient to focus on the Poincaré supersymmetries. To do so, we start by rewriting the solution in terms of a Calabi-Yau fourfold cone metric. We introduce coordinates for the AdS_4 space

$$\frac{1}{4}ds^2(AdS_4) = \frac{1}{4}\left(\frac{d\rho^2}{\rho^2} + \rho^2\eta_{\mu\nu}d\bar{\xi}^\mu d\bar{\xi}^\nu\right) = \frac{dr^2}{r^2} + r^4\eta_{\mu\nu}d\xi^\mu d\xi^\nu \tag{A.3}$$

with $\rho = r^2$, $\bar{\xi}^\mu = 2\xi^\mu$ and $\mu = 0, 1, 2$, and define the four-dimensional volume form $\mathrm{vol}_4 = r^5 d\xi^0 \wedge d\xi^1 \wedge d\xi^2 \wedge dr$. The $D = 11$ solution can then be recast in the form

$$\begin{aligned} ds^2 &= H^{-2/3}\eta_{\mu\nu}d\xi^\mu d\xi^\nu + H^{1/3}ds^2(C_8)\,, \\ G_4 &= d\xi^0 \wedge d\xi^1 \wedge d\xi^2 \wedge d(H^{-1}) \end{aligned} \tag{A.4}$$

where we have introduced the cone metric over the SE_7 space,

$$ds^2(C_8) = dr^2 + r^2 ds^2(SE_7)\,, \tag{A.5}$$

and $H = r^{-6}$ is harmonic on C_8. The eleven-dimensional volume form is then $\epsilon = H^{1/3}d\xi^0 \wedge d\xi^1 \wedge d\xi^2 \wedge \mathrm{vol}(C_8)$ where $\mathrm{vol}(C_8) = r^7 dr \wedge \mathrm{vol}(SE_7)$.

The Sasaki-Einstein structure (A.2) defines a unique Calabi-Yau structure on the cone given by the SU(4) invariant tensors

$$\begin{aligned} J_{CY} &= r dr \wedge \eta + r^2 J\,, \\ \Omega_{CY} &= r^3(dr + ir\eta) \wedge \Omega\,, \end{aligned} \tag{A.6}$$

determined by requiring the closure of J_{CY} and Ω_{CY} to be equivalent to $d\eta = 2J$ and $d\Omega = 4i\eta \wedge \Omega$. In particular, we then find

$$\mathrm{vol}(C_8) = \frac{1}{4!}J_{CY}^4 = \frac{1}{16}\Omega_{CY} \wedge \Omega^*_{CY}\ . \tag{A.7}$$

We now turn to the supersymmetry. We introduce a $D = 11$ orthonormal frame:

$$e^\mu = H^{-1/3}d\xi^\mu\,, \qquad e^{a+2} = H^{1/6}g^a\,, \quad a = 1, \ldots, 8\,, \tag{A.8}$$

where g^a is an orthonormal frame for the cone metric. Following the conventions of [34], by definition $\epsilon = e^0 \wedge e^1 \wedge \cdots \wedge e^{10}$ and so

$$\mathrm{vol}(C_8) = g^1 \wedge g^2 \wedge \cdots \wedge g^8\ . \tag{A.9}$$

We can then decompose the $D = 11$ gamma-matrices as

$$\begin{aligned} \Gamma_\mu &= \tau_\mu \otimes \gamma_{(8)}\,, \\ \Gamma_{a+2} &= \mathbf{1} \otimes \gamma_a\,, \qquad\qquad a = 1, \ldots, 8 \end{aligned} \tag{A.10}$$

with $\tau_{012} = 1$ and where $\gamma_{(8)} = \gamma_1\gamma_2\ldots\gamma_8$ is the chirality operator in $D = 8$. The $D = 11$ supersymmetry equations given in [34] are satisfied by a solution of the form (A.4) provided the supersymmetry transformation parameter satisfies the gamma-matrix projection condition

$$\Gamma_{012}\epsilon = \epsilon \quad \Leftrightarrow \quad \Gamma_{34\dots10}\epsilon = \epsilon \; . \tag{A.11}$$

More precisely, there are Poincaré Killing spinors of the form

$$\epsilon = H^{-1/6}\alpha\otimes\beta \,, \tag{A.12}$$

where α is a constant two-component Majorana spinor in $D=3$ and β is a 16-component Majorana-Weyl spinor in $D=8$ satisfying

$$\nabla_a\beta = 0 \,, \qquad \gamma_{(8)}\beta = \beta \; . \tag{A.13}$$

For there to be two independent solutions $\beta_{(i)}$ with $i=1,2$, the cone metric must be Calabi-Yau. In particular, the $\beta_{(i)}$ can be chosen to be orthogonal and the Calabi-Yau structure J_{CY} and Ω_{CY} can be written as bilinears in $\beta_{(i)}$. Specifically, one can choose a frame $\{g^a\}$ and spinor projections exactly as in appendix B of [58] such that

$$\begin{aligned} J_{CY} &= g^{12}+g^{34}+g^{56}+g^{78} \,, \\ \Omega_{CY} &= (g^1+ig^2)\wedge(g^3+ig^4)\wedge(g^5+ig^6)\wedge(g^7+ig^8) \; . \end{aligned} \tag{A.14}$$

Crucially, from (A.9), we see these satisfy the orientation relation (A.7). Thus the Calabi-Yau structure (A.6) on the cone C_8 defined by the Sasaki-Einstein structure (A.2) is indeed of the type required for the solution to be supersymmetric.

Note that if one takes the skew-whiffed solution where $G_4 = -\frac{3}{8}\,\mathrm{vol}(AdS_4)$, supersymmetry would then imply $\gamma_{(8)}\beta = -\beta$. This would in turn require a Calabi-Yau structure (J'_{CY}, Ω'_{CY}) on C_8 satisfying $\mathrm{vol}(C_8) = -\frac{1}{4!}J'^4_{CY} = -\frac{1}{16}\Omega'\wedge\Omega'^\star$. The structure defined by the Sasaki-Einstein manifold is not of this type, and hence the skew-whiffed solution is generically not supersymmetric.

B Details on the KK reduction

As discussed in the main text, our ansatz for the metric of $D=11$ supergravity is given by

$$ds^2 = ds_4^2 + e^{2U}ds^2(KE_6) + e^{2V}(\eta+A_1)\otimes(\eta+A_1) \tag{B.1}$$

while for the four-form we consider

$$\begin{aligned} G_4 = {} & f\mathrm{vol}_4 + H_3\wedge(\eta+A_1) + H_2\wedge J + H_1\wedge J\wedge(\eta+A_1) + 2hJ\wedge J \\ & +\sqrt{3}\left[\chi_1\wedge\Omega + \chi(\eta+A_1)\wedge\Omega + \text{c.c.}\right] \; . \end{aligned} \tag{B.2}$$

For the $D=11$ volume-form we choose $\epsilon = e^{6U+V}\,\mathrm{vol}_4\wedge\mathrm{vol}(KE_6)\wedge\eta$, where vol_4 is the $D=4$ volume form. In both $D=11$ and $D=4$ we use a mostly plus signature convention.

We now substitute this ansatz into the equations of motion of $D=11$ supergravity. The Bianchi identity $dG_4=0$ is satisfied provided

$$dH_3 = 0 \,, \tag{B.3}$$

$$dH_2 = 2H_3 + H_1 \wedge F_2\,, \tag{B.4}$$
$$H_1 = dh\,, \tag{B.5}$$
$$\chi_1 = -\frac{i}{4}D\chi\,, \tag{B.6}$$

where $F_2 \equiv dA_1$, $D\chi \equiv d\chi - 4iA_1\chi$ and we note that (B.3) follows from (B.4) and (B.5). Note that using (B.5) and (B.6) we can write the four-form as

$$\begin{aligned} G_4 = &\, f\,\mathrm{vol}_4 + H_3 \wedge (\eta + A_1) + H_2 \wedge J + dh \wedge J \wedge (\eta + A_1) + 2hJ \wedge J \\ &+ \sqrt{3}\left[\chi(\eta + A_1) \wedge \Omega - \frac{i}{4}D\chi \wedge \Omega + \text{c.c.}\right] . \end{aligned} \tag{B.7}$$

We solve equations (B.3) and (B.4) by introducing potentials B_2 and B_1 via

$$\begin{aligned} H_3 &= dB_2\,, \\ H_2 &= dB_1 + 2B_2 + hF_2\ . \end{aligned} \tag{B.8}$$

Similarly the equation of motion for the four-form, $d{*_{11}}G_4 + \frac{1}{2}G_4 \wedge G_4 = 0$, is also satisfied if

$$d\left(e^{6U-V} * H_3\right) - e^{6U+V} f F_2 + 6e^{2U+V} * H_2 + 12hH_2 + \frac{3i}{2}D\chi \wedge D\chi^* = 0 \tag{B.9}$$
$$d\left(e^{2U+V} * H_2\right) + 2dh \wedge H_2 + 4hH_3 = 0 \tag{B.10}$$
$$d\left(e^{2U-V} * dh\right) + e^{2U+V} * H_2 \wedge F_2 + H_2 \wedge H_2 + 4h\left(f + 4e^{-2U+V}\right)\mathrm{vol}_4 = 0 \tag{B.11}$$
$$d[e^{6U+V} f - 6(h^2 + |\chi|^2)] = 0 \tag{B.12}$$
$$D\left(e^V * D\chi\right) + iH_3 \wedge D\chi + 4\chi(f + 4e^{-V})\mathrm{vol}_4 = 0\ . \tag{B.13}$$

One can show that (B.10) can be obtained by acting with d on (B.9). We can solve (B.12) by setting

$$f = 6e^{-6U-V}(\pm 1 + h^2 + |\chi|^2) \tag{B.14}$$

where the constant factor of ± 6 (when $U = V = h = \chi = 0$) is chosen as a convenient normalisation. The upper sign corresponds to reducing to a $D = 4$ theory that contains the supesymmetric $AdS_4 \times SE_7$ solution of $D = 11$ supergravity while the lower sign corresponds to the skew-whiffed $AdS_4 \times SE_7$ solution, which generically does not preserve any supersymmetry.

Finally we consider the $D = 11$ Einstein equations:

$$R_{AB} = \frac{1}{12}G_{4\,AC_1C_2C_3}G_{4\,B}{}^{C_1C_2C_3} - \frac{1}{144}g_{AB}G_{4\,C_1C_2C_3C_4}G_4^{C_1C_2C_3C_4}\ . \tag{B.15}$$

To calculate the Ricci tensor for the $D = 11$ metric we use the orthonormal frame

$$\begin{aligned} \bar{e}^\alpha &= e^\alpha\ , && \alpha = 0,1,2,3\,, \\ \bar{e}^i &= e^U e^i\ , && i = 1,\ldots,6\,, \\ \bar{e}^7 &= e^V \hat{e}^7 \equiv e^V(\eta + A_1)\ . \end{aligned} \tag{B.16}$$

We then observe that the corresponding spin connection can be written

$$\begin{aligned}\bar{\omega}^{\alpha\beta} &= \omega^{\alpha\beta} - \frac{1}{2}e^{2V}F^{\alpha\beta}\hat{e}^7 \\ \bar{\omega}^{\alpha i} &= -e^U\partial^\alpha U e^i \\ \bar{\omega}^{\alpha 7} &= -e^V\partial^\alpha V\hat{e}^7 - \frac{1}{2}e^V F^\alpha{}_\beta e^\beta \\ \bar{\omega}^{ij} &= \omega^{ij} - e^{2V-2U}J^{ij}\hat{e}^7 \\ \bar{\omega}^{i7} &= -e^{V-U}J^i{}_j e^j\end{aligned} \tag{B.17}$$

After some computation we find that the components of the Ricci tensor, $\bar{R}_{AB}$, are given by

$$\begin{aligned}\bar{R}_{\alpha\beta} &= R_{\alpha\beta} - 6\left(\nabla_\beta\nabla_\alpha U + \partial_\alpha U\partial_\beta U\right) - \left(\nabla_\beta\nabla_\alpha V + \partial_\alpha V\partial_\beta V\right) - \frac{1}{2}e^{2V}F_{\alpha\gamma}F_\beta{}^\gamma \\ \bar{R}_{\alpha i} &= 0 \\ \bar{R}_{\alpha 7} &= -\frac{1}{2}e^{-2V-6U}\nabla_\gamma\left(e^{3V+6U}F^{\gamma\alpha}\right) \\ \bar{R}_{ij} &= \delta_{ij}\left[8e^{-2U} - 2e^{2V-4U} - \nabla_\gamma\nabla^\gamma U - 6\partial_\gamma U\partial^\gamma U - \partial_\gamma U\partial^\gamma V\right] \\ \bar{R}_{i7} &= 0 \\ \bar{R}_{77} &= 6e^{2V-4U} - \nabla_\gamma\nabla^\gamma V - 6\partial_\gamma U\partial^\gamma V - \partial_\gamma V\partial^\gamma V + \frac{1}{4}e^{2V}F_{\alpha\beta}F^{\alpha\beta}\end{aligned} \tag{B.18}$$

Using these results we find that the $D = 11$ Einstein equations (B.15) reduce to the following four equations in $D = 4$:

$$\begin{aligned}R_{\alpha\beta} = &\ 6\left(\nabla_\beta\nabla_\alpha U + \partial_\alpha U\partial_\beta U\right) + \left(\nabla_\beta\nabla_\alpha V + \partial_\alpha V\partial_\beta V\right) \\ &+\frac{3}{2}e^{-4U-2V}\left(\nabla_\alpha h\nabla_\beta h - \frac{1}{3}\eta_{\alpha\beta}\nabla_\lambda h\nabla^\lambda h\right) \\ &+\frac{3}{4}e^{-6U}\left[(D_\alpha\chi)(D_\beta\chi^*) + (D_\beta\chi)(D_\alpha\chi^*) - \frac{2}{3}\eta_{\alpha\beta}(D_\gamma\chi)(D^\gamma\chi^*)\right] \\ &-2\eta_{\alpha\beta}\left(e^{-8U}4h^2 + \frac{1}{6}f^2 + 4e^{-6U-2V}|\chi|^2\right) + \frac{1}{2}e^{2V}F_{\alpha\gamma}F_\beta{}^\gamma \\ &+\frac{1}{4}e^{-2V}\left(H_{\alpha\lambda\mu}H_\beta{}^{\lambda\mu} - \frac{1}{9}\eta_{\alpha\beta}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right) \\ &+\frac{3}{2}e^{-4U}\left(H_{\alpha\lambda}H_\beta{}^\lambda - \frac{1}{6}\eta_{\alpha\beta}H_{\lambda\mu}H^{\lambda\mu}\right)\end{aligned} \tag{B.19}$$

$$\nabla_\gamma\left(e^{3V+6U}F^\gamma{}_\alpha\right) = \frac{1}{6}e^{6U+V}f\epsilon_{\alpha\beta\gamma\delta}H^{\beta\gamma\delta} + 3e^{2U+V}H_{\alpha\beta}\nabla^\beta h + 6ie^V\left[\chi^* D_\alpha\chi - \chi D_\alpha\chi^*\right] \tag{B.20}$$

$$\begin{aligned}\nabla_\gamma\nabla^\gamma U + 6\partial_\gamma U\partial^\gamma U + \partial_\gamma U\partial^\gamma V + \frac{1}{4}e^{-6U}(D_\gamma\chi)(D^\gamma\chi^*) - \frac{1}{36}e^{-2V}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} \\ -8e^{-2U} + 2e^{2V-4U} + 8e^{-8U}h^2 + \frac{1}{6}f^2 + 4e^{-6U-2V}|\chi|^2 = 0\end{aligned} \tag{B.21}$$

$$\begin{aligned}\nabla_\gamma\nabla^\gamma V + 6\partial_\gamma U\partial^\gamma V + \partial_\gamma V\partial^\gamma V + e^{-4U-2V}\nabla_\lambda h\nabla^\lambda h - \frac{1}{2}e^{-6U}(D_\gamma\chi)(D^\gamma\chi^*) \\ -6e^{2V-4U} - 8e^{-8U}h^2 + \frac{1}{6}f^2 + 16e^{-6U-2V}|\chi|^2 \\ -\frac{1}{4}e^{2V}F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{18}e^{-2V}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} - \frac{1}{4}e^{-4U}H_{\alpha\beta}H^{\alpha\beta} = 0\end{aligned} \tag{B.22}$$

JHEP04(2009)102

All of the dependence on the internal SE_7 space has dropped out. In particular any solution to the $D = 4$ field equations (B.3)–(B.6), (B.9)–(B.13), (B.19)–(B.22) gives rise to an exact solution to the equations of motion of $D = 11$ supergravity. Thus the KK ansatz (B.1), (B.7) is consistent.

C $\quad N = 2$ supergravity

The bosonic part of the general gauged $\mathcal{N} = 2$ supergravity action coupled to vector and hypermultiplets is given by [16, 59]

$$S = \int \frac{1}{2} R * 1 + g_{i\bar{j}} Dt^i \wedge * D\bar{t}^{\bar{j}} + h_{uv} Dq^u \wedge * Dq^v + \frac{1}{2} \operatorname{Im} \mathcal{N}_{IJ} F^I \wedge * F^J + \frac{1}{2} \operatorname{Re} \mathcal{N}_{IJ} F^I \wedge F^J - V \,. \tag{C.1}$$

Here t^i, $i = 1, \dots, n_V$ are the complex scalar fields in the n_V vector multiplets parameterizing a special Kähler manifold with metric $g_{i\bar{j}}$, while q^u, $u = 1, \dots, 4n_H$, are the real scalar fields in the n_H hypermultiplets parameterizing a quaternionic manifold with metric h_{uv}. The two-forms $F^I = dA^I$ with $I = 0, 1, \dots, n_V$ are the gauge field strengths for the vector multiplet and graviphoton potentials A^I. In the gauged theory

$$D_\mu t^i = \partial_\mu t^i - k^i_I A^I_\mu \,, \qquad D_\mu q^u = \partial_\mu q^u - k^u_I A^I_\mu \,, \tag{C.2}$$

where k^i_I and k^u_I are Killing vectors on the special Kähler and quaternionic manifolds. For the theories appearing in this paper $k^i_I = 0$.

The metric on the special Kähler manifold and the gauge kinetic terms can both be written in terms of a holomorphic prepotential $\mathcal{F}(X)$ where $X^I(t)$ are homogeneous coordinates on the manifold and which is a homogeneous function of degree two. Explicitly the Kähler potential and $\mathcal{N}_{IJ}$ matrix are given by

$$\begin{aligned} K_V &= -\log\left(i\bar{X}^I \mathcal{F}_I - iX^I \bar{\mathcal{F}}_I\right) \,, \\ \mathcal{N}_{IJ} &= \bar{\mathcal{F}}_{IJ} + 2i \frac{(\operatorname{Im} \mathcal{F}_{IK})(\operatorname{Im} \mathcal{F}_{JL}) X^K X^L}{(\operatorname{Im} \mathcal{F}_{AB}) X^A X^B} \,, \end{aligned} \tag{C.3}$$

with $\mathcal{F}_I = \partial_I \mathcal{F}$ and $\mathcal{F}_{IJ} = \partial_I \partial_J \mathcal{F}$. Under symplectic transformations acting on the gauge fields F^I and the generalised duals $G_I = \partial \mathcal{L} / \partial A^I$, where $\mathcal{L}$ is the scalar Lagrangian for the supergravity action (C.1), one has

$$\begin{pmatrix} F^I \\ G_I \end{pmatrix} \mapsto \begin{pmatrix} \tilde{F}^I \\ \tilde{G}_I \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F^I \\ G_I \end{pmatrix} \,, \tag{C.4}$$

where $A^T D - C^T B = 1$, $A^T C = C^T A$ and $B^T D = D^T B$. The $(X^I, \mathcal{F}_I)$ coordinates and $\mathcal{N}_{IJ}$ then transform as

$$\begin{gathered} \begin{pmatrix} X^I \\ \mathcal{F}_I \end{pmatrix} \mapsto \begin{pmatrix} \tilde{X}^I \\ \tilde{\mathcal{F}}_I \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X^I \\ \mathcal{F}_I \end{pmatrix} \,, \\ \mathcal{N} \mapsto \tilde{\mathcal{N}} = (C + D\mathcal{N})(A + B\mathcal{N})^{-1} \,. \end{gathered} \tag{C.5}$$

The quaternionic manifold has $\mathrm{SU}(2)\times\mathrm{Sp}(2n_H)$ special holonomy, so, as in for example [60], one can introduce vielbeins $V^{A\alpha}$ where $A=1,2$ and $\alpha=1,\dots,n_H$ such that $h_{uv}=V_u^{A\alpha}V_v^{B\beta}\epsilon_{AB}\mathbb{C}_{\alpha\beta}$ where $\epsilon_{12}=-1$ and $\mathbb{C}_{\alpha\beta}$ is the constant symplectic form for $\mathrm{Sp}(n_H)$. This defines SU(2) and $\mathrm{Sp}(n_V)$ connections via $dV^{Aa}+\omega^A_{\ B}\wedge V^{Ba}+\Delta^a_{\ b}\wedge V^{Ab}=0$. The triplet of Kähler forms can then be written as

$$K=K^x\left(-\frac{i}{2}\sigma^x\right)=-\frac{1}{2}\left(d\omega+\omega\wedge\omega\right), \tag{C.6}$$

where σ^x with $x=1,2,3$ are the Pauli matrices. The corresponding complex structures $(J^x)^u_{\ v}=h^{uw}(K^x)_{wv}$ then satisfy the quaternion algebra.

Given the Killing vectors k_I^u one can then introduce triplets of Killing prepotentials $P_I=P_I^x\left(-\frac{i}{2}\sigma^x\right)$ satisfying

$$i_{k_I}K=dP_I+[\omega,P_I]\ . \tag{C.7}$$

If the gauging is only in the hypermultiplet sector then the potential V in the action (C.1) is given by

$$V=e^{K_V}X^I\bar{X}^J\left(4h_{uv}k_I^uk_J^v\right)-\left(\frac{1}{2}(\mathrm{Im}\mathcal{N})^{-1IJ}+4e^{K_V}X^I\bar{X}^J\right)P_I^xP_J^x\ . \tag{C.8}$$

It is well-known that the universal hypermultiplet parameterizes a SU(2, 1)/U(2) coset. One can identify the particular quaternionic geometry as follows [61]. The metric h_{uv} can be written as

$$h_{uv}dq^udq^v=\frac{1}{4\rho^2}d\rho^2+\frac{1}{4\rho^2}\left[d\sigma-i(\xi d\bar{\xi}-\bar{\xi}d\xi)\right]^2+\frac{1}{\rho}d\xi d\bar{\xi}\,, \tag{C.9}$$

which has Ricci tensor equal to minus six times the metric. Introducing the one-forms

$$\alpha=\frac{d\xi}{\sqrt{\rho}},\qquad \beta=\frac{1}{2\rho}\left(d\rho+id\sigma+\xi d\bar{\xi}-\bar{\xi}d\xi\right) \tag{C.10}$$

one can write

$$V^{A\alpha}=\frac{1}{\sqrt{2}}\begin{pmatrix}\alpha & \bar{\beta}\\ \beta & -\bar{\alpha}\end{pmatrix}^{A\alpha} \tag{C.11}$$

and $h_{uv}=\epsilon_{\alpha\beta}\mathbb{C}_{AB}V_u^{A\alpha}V_v^{B\beta}$ with the constant symplectic form $\mathbb{C}$ having components $\mathbb{C}_{12}=1$. We also find

$$\omega^A_{\ B}=\begin{pmatrix}\frac{1}{4}(\beta-\bar{\beta}) & -\alpha\\ \bar{\alpha} & -\frac{1}{4}(\beta-\bar{\beta})\end{pmatrix}^A_{\ \ B},\quad \Delta^\alpha_{\ \beta}=\begin{pmatrix}-\frac{3}{4}(\beta-\bar{\beta}) & 0\\ 0 & \frac{3}{4}(\beta-\bar{\beta})\end{pmatrix}^\alpha_{\ \ \beta}. \tag{C.12}$$

and

$$K^A_{\ B}=\begin{pmatrix}\frac{1}{2}(\alpha\wedge\bar{\alpha}-\beta\wedge\bar{\beta}) & \alpha\wedge\bar{\beta}\\ \beta\wedge\bar{\alpha} & -\frac{1}{2}(\alpha\wedge\bar{\alpha}-\beta\wedge\bar{\beta})\end{pmatrix}^A_{\ \ B}. \tag{C.13}$$

References

[1] J.P. Gauntlett and O. Varela, *Consistent Kaluza-Klein reductions for general supersymmetric AdS solutions*, *Phys. Rev.* **D 76** (2007) 126007 [arXiv:0707.2315] [SPIRES].

[2] M.J. Duff and C.N. Pope, *Consistent truncations in Kaluza-Klein theories*, *Nucl. Phys.* **B 255** (1985) 355 [SPIRES].

[3] A. Buchel and J.T. Liu, *Gauged supergravity from type IIB string theory on $Y(p,q)$ manifolds*, *Nucl. Phys.* **B 771** (2007) 93 [hep-th/0608002] [SPIRES].

[4] J.P. Gauntlett, E. O Colgain and O. Varela, *Properties of some conformal field theories with M-theory duals*, *JHEP* **02** (2007) 049 [hep-th/0611219] [SPIRES].

[5] J.P. Gauntlett and O. Varela, $D = 5$ SU(2) × U(1) *gauged supergravity from* $D = 11$ *supergravity*, *JHEP* **02** (2008) 083 [arXiv:0712.3560] [SPIRES].

[6] T.T. Tsikas, *Consistent truncations of chiral* $N = 2$ $D = 10$ *supergravity on the round five sphere*, *Class. Quant. Grav.* **3** (1986) 733 [SPIRES].

[7] M.J. Duff, B.E.W. Nilsson, C.N. Pope and N.P. Warner, *On the consistency of the Kaluza-Klein ansatz*, *Phys. Lett.* **B 149** (1984) 90 [SPIRES].

[8] M. Cvetič et al., *Embedding AdS black holes in ten and eleven dimensions*, *Nucl. Phys.* **B 558** (1999) 96 [hep-th/9903214] [SPIRES].

[9] H. Lü, C.N. Pope and T.A. Tran, *Five-dimensional* $N = 4$, SU(2) × U(1) *gauged supergravity from type IIB*, *Phys. Lett.* **B 475** (2000) 261 [hep-th/9909203] [SPIRES].

[10] M. Cvetič, H. Lü, C.N. Pope, A. Sadrzadeh and T.A. Tran, *Consistent* SO(6) *reduction of type IIB supergravity on* S^5, *Nucl. Phys.* **B 586** (2000) 275 [hep-th/0003103] [SPIRES].

[11] A. Khavaev, K. Pilch and N.P. Warner, *New vacua of gauged* $N = 8$ *supergravity in five dimensions*, *Phys. Lett.* **B 487** (2000) 14 [hep-th/9812035] [SPIRES].

[12] B. de Wit and H. Nicolai, *The consistency of the* S^7 *truncation in* $D = 11$ *supergravity*, *Nucl. Phys.* **B 281** (1987) 211 [SPIRES].

[13] J. Maldacena, D. Martelli and Y. Tachikawa, *Comments on string theory backgrounds with non-relativistic conformal symmetry*, *JHEP* **10** (2008) 072 [arXiv:0807.1100] [SPIRES].

[14] M.S. Bremer, M.J. Duff, H. Lu, C.N. Pope and K.S. Stelle, *Instanton cosmology and domain walls from M-theory and string theory*, *Nucl. Phys.* **B 543** (1999) 321 [hep-th/9807051] [SPIRES].

[15] J.T. Liu and H. Sati, *Breathing mode compactifications and supersymmetry of the brane-world*, *Nucl. Phys.* **B 605** (2001) 116 [hep-th/0009184] [SPIRES].

[16] J. Louis and A. Micu, *Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, *Nucl. Phys.* **B 635** (2002) 395 [hep-th/0202168] [SPIRES].

[17] U. Theis and S. Vandoren, $N = 2$ *supersymmetric scalar-tensor couplings*, *JHEP* **04** (2003) 042 [hep-th/0303048] [SPIRES].

[18] G. Dall'Agata, R. D'Auria, L. Sommovigo and S. Vaula, $D = 4$, $N = 2$ *gauged supergravity in the presence of tensor multiplets*, *Nucl. Phys.* **B 682** (2004) 243 [hep-th/0312210] [SPIRES]; R. D'Auria, L. Sommovigo and S. Vaula, $N = 2$ *supergravity Lagrangian coupled to tensor multiplets with electric and magnetic fluxes*, *JHEP* **11** (2004) 028 [hep-th/0409097] [SPIRES].

JHEP04(2009)102

[19] S.M. Kuzenko, *On massive tensor multiplets*, *JHEP* **01** (2005) 041 [hep-th/0412190] [SPIRES]; *On $N = 2$ supergravity and projective superspace: dual formulations*, *Nucl. Phys.* **B 810** (2009) 135 [arXiv:0807.3381] [SPIRES].

[20] R. D'Auria and S. Ferrara, *Dyonic masses from conformal field strengths in D even dimensions*, *Phys. Lett.* **B 606** (2005) 211 [hep-th/0410051] [SPIRES];
J. Louis and W. Schulgin, *Massive tensor multiplets in $N = 1$ supersymmetry*, *Fortsch. Phys.* **53** (2005) 235 [hep-th/0410149] [SPIRES];
U. Theis, *Masses and dualities in extended Freedman-Townsend models*, *Phys. Lett.* **B 609** (2005) 402 [hep-th/0412177] [SPIRES].

[21] For a review see M. Grana, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003] [SPIRES].

[22] O. Aharony, M. Berkooz, J. Louis and A. Micu, *Non-Abelian structures in compactifications of M-theory on seven-manifolds with* SU(3) *structure*, *JHEP* **09** (2008) 108 [arXiv:0806.1051] [SPIRES].

[23] T. House and A. Micu, *M-theory compactifications on manifolds with G_2 structure*, *Class. Quant. Grav.* **22** (2005) 1709 [hep-th/0412006] [SPIRES].

[24] J.P. Gauntlett, S. Kim, O. Varela and D. Waldram, to appear.

[25] D.T. Son, *Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry*, *Phys. Rev.* **D 78** (2008) 046003 [arXiv:0804.3972] [SPIRES].

[26] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, *Phys. Rev. Lett.* **101** (2008) 061601 [arXiv:0804.4053] [SPIRES].

[27] C.P. Herzog, M. Rangamani and S.F. Ross, *Heating up Galilean holography*, *JHEP* **11** (2008) 080 [arXiv:0807.1099] [SPIRES].

[28] A. Adams, K. Balasubramanian and J. McGreevy, *Hot spacetimes for cold atoms*, *JHEP* **11** (2008) 059 [arXiv:0807.1111] [SPIRES].

[29] M.J. Duff, B.E.W. Nilsson and C.N. Pope, *The criterion for vacuum stability in Kaluza-Klein supergravity*, *Phys. Lett.* **B 139** (1984) 154 [SPIRES].

[30] F. Denef and S.A. Hartnoll, *Landscape of superconducting membranes*, arXiv:0901.1160 [SPIRES].

[31] S.S. Gubser, *Breaking an Abelian gauge symmetry near a black hole horizon*, *Phys. Rev.* **D 78** (2008) 065034 [arXiv:0801.2977] [SPIRES].

[32] S.A. Hartnoll, C.P. Herzog and G.T. Horowitz, *Building a holographic superconductor*, *Phys. Rev. Lett.* **101** (2008) 031601 [arXiv:0803.3295] [SPIRES].

[33] S.A. Hartnoll, C.P. Herzog and G.T. Horowitz, *Holographic superconductors*, *JHEP* **12** (2008) 015 [arXiv:0810.1563] [SPIRES].

[34] J.P. Gauntlett and S. Pakis, *The geometry of $D = 11$ Killing spinors*, *JHEP* **04** (2003) 039 [hep-th/0212008] [SPIRES].

[35] J.A. Minahan and R.C. Warner, *Stuckelberg revisited*, Florida University Report No. UFIFT-HEP-89-15, unpublished [SPIRES].

[36] Y. Takahashi and R. Palmer, *Gauge-independent formulation of a massive field with spin one*, *Phys. Rev.* **D 1** (1970) 2974 [SPIRES].

[37] F. Quevedo and C.A. Trugenberger, *Phases of antisymmetric tensor field theories*, *Nucl. Phys.* **B 501** (1997) 143 [hep-th/9604196] [SPIRES].

[38] D. Fabbri, P. Fré, L. Gualtieri and P. Termonia, *M-theory on $AdS_4 \times M^{111}$: the complete $Osp(2|4) \times \mathrm{SU}(3) \times \mathrm{SU}(2)$ spectrum from harmonic analysis*, *Nucl. Phys.* **B 560** (1999) 617 [hep-th/9903036] [SPIRES].

[39] M. Günaydin, S. McReynolds and M. Zagermann, *The R-map and the coupling of $N = 2$ tensor multiplets in 5 and 4 dimensions*, *JHEP* **01** (2006) 168 [hep-th/0511025] [SPIRES].

[40] A. Donos and J.P. Gauntlett, *Supersymmetric solutions for non-relativistic holography*, *JHEP* **03** (2009) 138 [arXiv:0901.0818] [SPIRES].

[41] P. Fré', L. Gualtieri and P. Termonia, *The structure of $N = 3$ multiplets in AdS_4 and the complete $Osp(3|4) \times \mathrm{SU}(3)$ spectrum of M-theory on $AdS_4 \times N(0,1,0)$*, *Phys. Lett.* **B 471** (1999) 27 [hep-th/9909188] [SPIRES].

[42] A. Casher, F. Englert, H. Nicolai and M. Rooman, *The mass spectrum of supergravity on the round seven sphere*, *Nucl. Phys.* **B 243** (1984) 173 [SPIRES].

[43] M.J. Duff, B.E.W. Nilsson and C.N. Pope, *Kaluza-Klein supergravity*, *Phys. Rept.* **130** (1986) 1 [SPIRES].

[44] M.J. Duff, C.N. Pope and K.S. Stelle, *Consistent interacting massive spin-2 requires an infinity of states*, *Phys. Lett.* **B 223** (1989) 386 [SPIRES].

[45] H.J. Kim, L.J. Romans and P. van Nieuwenhuizen, *The mass spectrum of chiral $N = 2$ $D = 10$ supergravity on S^5*, *Phys. Rev.* **D 32** (1985) 389 [SPIRES].

[46] S.S. Gubser, A. Hashimoto, I.R. Klebanov and M. Krasnitz, *Scalar absorption and the breaking of the world volume conformal invariance*, *Nucl. Phys.* **B 526** (1998) 393 [hep-th/9803023] [SPIRES].

[47] K.A. Intriligator, *Maximally supersymmetric RG flows and AdS duality*, *Nucl. Phys.* **B 580** (2000) 99 [hep-th/9909082] [SPIRES].

[48] U.H. Danielsson, A. Guijosa, M. Kruczenski and B. Sundborg, *D3-brane holography*, *JHEP* **05** (2000) 028 [hep-th/0004187] [SPIRES].

[49] H. Nastase, D. Vaman and P. van Nieuwenhuizen, *Consistent nonlinear KK reduction of $11D$ supergravity on $AdS_7 \times S_4$ and self-duality in odd dimensions*, *Phys. Lett.* **B 469** (1999) 96 [hep-th/9905075] [SPIRES].

[50] H. Nastase, D. Vaman and P. van Nieuwenhuizen, *Consistency of the $AdS_7 \times S_4$ reduction and the origin of self-duality in odd dimensions*, *Nucl. Phys.* **B 581** (2000) 179 [hep-th/9911238] [SPIRES].

[51] R.G. Leigh and M. Rozali, *The large-N limit of the $(2,0)$ superconformal field theory*, *Phys. Lett.* **B 431** (1998) 311 [hep-th/9803068] [SPIRES].

[52] P. van Nieuwenhuizen, *The complete mass spectrum of $D = 11$ supergravity compactified on S_4 and a general mass formula for arbitrary cosets M_4*, *Class. Quant. Grav.* **2** (1985) 1 [SPIRES].

[53] M. Günaydin, P. van Nieuwenhuizen and N.P. Warner, *General construction of the unitary representations of anti-de Sitter superalgebras and the spectrum of the S^4 compactification of eleven-dimensional supergravity*, *Nucl. Phys.* **B 255** (1985) 63 [SPIRES].

[54] H. Lin, O. Lunin and J.M. Maldacena, *Bubbling AdS space and 1/2 BPS geometries*, *JHEP* **10** (2004) 025 [hep-th/0409174] [SPIRES].

[55] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Supersymmetric AdS_5 solutions of M-theory*, *Class. Quant. Grav.* **21** (2004) 4335 [hep-th/0402153] [SPIRES].

[56] J.P. Gauntlett, O.A.P. Mac Conamhna, T. Mateos and D. Waldram, *AdS spacetimes from wrapped M5 branes*, *JHEP* **11** (2006) 053 [hep-th/0605146] [SPIRES].

[57] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Supersymmetric AdS_5 solutions of type IIB supergravity*, *Class. Quant. Grav.* **23** (2006) 4693 [hep-th/0510125] [SPIRES].

[58] J.P. Gauntlett, D. Martelli and D. Waldram, *Superstrings with intrinsic torsion*, *Phys. Rev.* **D 69** (2004) 086002 [hep-th/0302158] [SPIRES].

[59] For a complete review see L. Andrianopoli et al., *$N = 2$ supergravity and $N = 2$ super Yang-Mills theory on general scalar manifolds: symplectic covariance, gaugings and the momentum map*, *J. Geom. Phys.* **23** (1997) 111 [hep-th/9605032] [SPIRES].

[60] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Heterotic M-theory in five dimensions*, *Nucl. Phys.* **B 552** (1999) 246 [hep-th/9806051] [SPIRES].

[61] A. Strominger, *Loop corrections to the universal hypermultiplet*, *Phys. Lett.* **B 421** (1998) 139 [hep-th/9706195] [SPIRES].

JHEP04(2009)102